

Final – 100 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. No cell phones. Do not use your own scratch paper.

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature KEY

Problem 1 (40 Points)

Consider the following simple specification that tests for final exam performance as a function of first midterm performance and student cohort:

$$\ln(\text{final}) = \beta_0 + \beta_1 \ln(\text{mt1}) + \beta_2 \text{senior} + u$$

Here, *final* is the score on the final exam (out of 100), *mt1* is the score on midterm 1 (out of 100), and *senior* is a dummy variable that takes on a value of 1 if student is a senior, and 0 otherwise. The results from estimating this equation are below:

Source	SS	df	MS	
Model	.757668998	2	.378834499	Number of obs = 390
Residual	9.30424195	387	.024041969	F(2, 387) = 15.76
				Prob > F = 0.0000
				R-squared = 0.0753
				Adj R-squared = 0.0705
				Root MSE = .15505

ln_m4	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ln_m1	.1493374	.0282414			#####
senior	-.0251903	.0157726			#####
_cons	3.744924	.1251634			#####

a.) Please interpret the coefficient on *senior*. (5 points)

Holding *mt1* constant, seniors do 2.5% worse than non-seniors. + 5

b.) Does the regression tell us anything about the dependent variable? Please test this hypothesis at the 95% level, stating your null and alternative hypothesis. (5 points)

$$H_0: B_1 = 0, B_2 = 0 \quad +2$$

$$H_1: H_0 \text{ not true}$$

$$F_{\text{stat}} = 15.76$$

$$F_{\text{crit}} = 3 \quad +2$$

$$F_{\text{stat}} > F_{\text{crit}}$$

Reject $H_0!$

Regression tells us meaningful info $+1$

c.) I claim that "senioritis" is real, and that seniors have a significantly different score on the final exam than non-seniors. What is the probability that I'm wrong? (10 Points)

$$P_{\text{value}} = 2(1 - \Pr(T < |t_{\text{stat}}|)) \quad +4$$

$$= 2(1 - \Pr(T < 1.60)) \quad +4$$

$$= 2(1 - 0.9452) = 0.111 \quad +2$$

d.) Please construct and interpret a 99% confidence interval for the coefficient on $\ln(\text{mtl})$. (10 Points)

$$CI \quad B_1 \pm t_{99} \cdot se B_1 \quad +5$$

$$\Rightarrow 0.1493 - 2.575 \cdot 0.01577 \leq B \leq 0.1493 + 2.575 \cdot 0.01577$$

$$0.1087 \leq B \leq 0.1899$$

A 1% improvement in $\ln(\text{mtl})$ yields between a 0.1087 and 0.1899% improvement in the final with 99% confidence. $+5$

e.) Suppose my definition of senioritis is incorrect, where senioritis is instead defined as the relationship between the first exam and the final and how that depends on cohort. Hence, I estimate the following:

$$\ln(\text{final}) = \beta_0 + \beta_1 \ln(\text{mt1}) + \beta_2 \text{senior} + \beta_3 \ln(\text{mt1}) \cdot \text{senior} + u$$

The results from running this regression are below:

Source	SS	df	MS	
Model	.888890262	3	.296296754	Number of obs = 390
Residual	9.17302068	386	.023764302	F(3, 386) = 12.47
				Prob > F = 0.0000
				R-squared = 0.0883
				Adj R-squared = 0.0813
				Root MSE = .15416

ln_m4	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
ln_m1	.2263322	.0431505			#####
senior	.5627498	.2506945			#####
ln_m1_senior	-.1335334	.0568264			#####
_cons	3.404939	.190836			#####

Please interpret the coefficient on the interaction term, and test whether it is significantly different from zero at the 96% level. (10 Points)

A 1% improvement in mt1 yields a 0.13% lower improvement for seniors compared to non-seniors. +5

$$H_0: \beta_3 = 0$$

$$H_A: \beta_3 \neq 0 \quad (+2)$$

$$t_{\text{stat}} = \frac{-0.1335 - 0}{0.0508} = -2.35$$

$$t_{\text{crit}} = 2.05 \quad t_{\text{stat}} > t_{\text{crit}} \quad (+2)$$

Reject $H_0!$

β_3 is significantly different from zero.

+1

Problem 2 (60 Points)

a.) For this problem, we wish to study the effects of location and education on wage outcomes. We begin by estimating:

$$\log(\text{wage}) = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{south} + \beta_3 \text{urban} + \beta_4 \text{south} \cdot \text{urban} + u$$

Here, *wage* is monthly wage, *educ* is years of education, and *south* and *urban* are dummy variables identifying respondents that live in the south and in metropolitan areas, respectively. The results from this regression are below:

Source	SS	df	MS	Number of obs =	935
Model	25.4347562	4	6.35868906	F(4, 930) =	42.17
Residual	140.221538	930	.150775847	Prob > F =	0.0000
				R-squared =	0.1535
				Adj R-squared =	0.1499
Total	165.656294	934	.177362199	Root MSE =	.3883

lwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
educ	.0552406	.0058354	#####	#####	#####
south	-.0819054	.048621	#####	#####	#####
urban	.1840049	.0362901	#####	#####	#####
south_urban	-.0724756	.0583629	#####	#####	#####
_cons	5.946937	.0853035	#####	#####	#####

Please interpret the coefficient on the interaction between *south* and *urban* in terms of the returns to living in a city. Please test whether it is significantly different from zero at the 90% level. (10 Points)

The returns to living in a city are 7.2% + 5 lower for southern residents compared to northern residents.

$H_0: \beta_4 = 0$

$H_A: \beta_4 \neq 0$ +2

$t_{stat} = \frac{-0.0724 - 0}{0.0584} = -1.24$

$t_{crit} = 1.64$ +2

$|t_{stat}| < t_{crit} \Rightarrow$ Fail to reject H_0

β_4 is not significantly different from zero. +1

5.) You're unhappy with the regression in 'a' since there aren't non-linear effects of education. Instead, we estimate the following:

$$\log(\text{wage}) = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{south} + \beta_3 \text{urban} + \beta_4 \text{south} \cdot \text{urban} + \beta_5 \text{educ}^2 + \beta_6 \text{educ}^3 + u$$

The results are presented below:

Source	SS	df	MS	Number of obs = 935	
Model	25.8219974	6	4.30366623	F(6, 928) =	28.56
Residual	139.834297	928	.15068351	Prob > F =	0.0000
				R-squared =	0.1559
				Adj R-squared =	0.1504
				Root MSE =	.38818

lwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
educ	-.6084502	.5193148			#####
south	-.0826128	.0486587			#####
urban	.1833712	.0362819			#####
south_urban	-.0742973	.058378			#####
educ2	.0512586	.0380414			#####
educ3	-.0012899	.0009178			#####
_cons	8.751734	2.340047			#####

Which regression is preferred, the regression in '2a' or the regression in '2b'? If a hypothesis test is warranted, test this hypothesis at the 95% level, stating your null and alternative hypotheses. If not, provide other evidence for your answer. (10 Points)

F Test! $H_0: \beta_5 = 0, \beta_6 = 0$ + 2
 $H_A: H_0$ not true

$SSR_{ur} = 139.8$ $g = 2$
 $SSR_R = 140.22$ $df = 928$
 + 2

$$F_{stat} = \frac{\frac{140.22 - 139.8}{2}}{\frac{139.8}{928}} = 1.393$$

+ 11

$F_{crit} = 3$

$F_{stat} < F_{crit} \Rightarrow$ Fail to reject H_0

\Rightarrow We prefer 2a + 2

2 points if Adj R²

c.) Using the regression estimates from 'b', please calculate the marginal return of additional education for a respondent that currently has 12 years of education (this answer is a number, not an equation). (10 Points)

$$\frac{\partial \ln \text{wage}}{\partial \text{educ}} = \beta_1 + 2\beta_2 \text{educ} + 3\beta_3 \text{educ}^2 + 5$$

$$= -0.608 + 2(0.0513) \cdot 12 + 3 \cdot (-0.00129) \cdot (12)^2$$

$$\boxed{= 0.0659} + 3$$

or, ~~an~~ an additional year of education yields a 6.59% increase in the wage.

+2

d.) You're now unhappy with 'b' as well, and instead recommend the following:

$$\log(\text{wage}) = \beta_0 + \beta_1 \log(\text{educ}) + \beta_2 \text{south} + \beta_3 \text{south} + \beta_4 \text{south} \cdot \text{urban} + u$$

The results are presented below:

Source	SS	df	MS			
Model	25.4718488	4	6.3679622	Number of obs =	935	
Residual	140.184445	930	.150735963	F(4, 930) =	42.25	
				Prob > F =	0.0000	
				R-squared =	0.1538	
				Adj R-squared =	0.1501	
Total	165.656294	934	.177362199	Root MSE =	.38825	

l wage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
leduc	.76265	.0804417		#####	
south	-.0805032	.0486296		#####	
urban	.1840493	.0362853		#####	
south_urban	-.0738845	.0583648		#####	
_cons	4.717399	.2112056		#####	

Which regression is preferred, the regression in '2b' or the regression in '2d'? If a hypothesis test is warranted, test this hypothesis at the 95% level, stating your null and alternative hypotheses. If not, provide other evidence for your answer. (10 Points)

Non-nested D: $Adj R^2 = 0.1501 + 1$

B: $Adj R^2 = 0.1504 + 1$

Prefer 2B + 2

2 points if F-test

e.) Suppose that we switch back to something more traditional – the returns to education by location:

$$\log(\text{wage}) = \beta_0 + \beta_1 \log(\text{educ}) + \beta_2 \text{urban} + \beta_3 \log(\text{educ}) \cdot \text{urban} + u$$

Please derive an estimating equation that allows us to estimate (with standard error) the elasticity of wages with respect to education for an urban resident. (10 Points)

$$\frac{d\text{wage}}{d\text{educ}} = \Theta = \beta_1 + \beta_3 \text{urban} + \text{df}$$

$$\text{urban} = 1 \Rightarrow \Theta = \beta_1 + \beta_3$$

$$\Rightarrow \beta_3 = \Theta - \beta_1 + 2$$

$$\log(\text{wage}) = \beta_0 + \beta_1 \log(\text{educ}) + \beta_2 \text{urban} + (\Theta - \beta_1) \log(\text{educ}) \cdot \text{urban} + u + 2$$

$$\log(\text{wage}) = \beta_0 + \beta_1 \log(\text{educ})(1 - \text{urban}) + \beta_2 \text{urban} + \Theta \log(\text{educ}) \cdot \text{urban} + u$$

+ 2

f.) The results from running the regression in 'd' are presented below:

Source	SS	df	MS	
Model	22.0703681	3	7.35678937	Number of obs = 935
Residual	143.585926	931	.154227633	F(3, 931) = 47.70
				Prob > F = 0.0000
				R-squared = 0.1332
				Adj R-squared = 0.1304
Total	165.656294	934	.177362199	Root MSE = .39272

l wage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
leduc	.5802011	.1528491	#####	#####	#####
urban	-.5854885	.464816	#####	#####	#####
leduc_urban	.2932095	.1800985	#####	#####	#####
_cons	5.151897	.3933628	#####	#####	#####

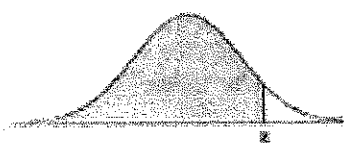
Please interpret the coefficient on $\log(\text{educ})$, and construct a 95% confidence interval. (10 Points)

For rural residents, a 1% increase in education yields a 0.58% increase in wages. +5

$$0.58 - 0.153 \cdot 1.96 < B_1 < 0.58 + 0.153 \cdot 1.96$$

$$\underline{0.28 < B_1 < 0.88} \quad +5$$

Have a nice holiday!!!



Normal Distribution from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990